Routing Directions: Keeping it Fast and Simple

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ABSTRACT

The problem of providing meaningful routing directions over road networks is of great importance. In many real-life cases, the fastest route may not be the ideal choice for providing directions in written/spoken text, or for an unfamiliar neighborhood, or in cases of emergency. Rather, it is often more preferable to offer "simple" directions that are easy to memorize, explain, understand or follow. However, there exist cases where the simplest route is considerably longer than the fastest. This paper tries to address this issue, by finding near-simplest routes which are as short as possible and near-fastest routes which are as simple as possible. Particularly, we focus on efficiency, and propose novel algorithms, which are theoretically and experimentally shown to be significantly faster than existing approaches.

Categories and Subject Descriptors

H.2.8 [**Database Management**]: Database Applications—*Spatial databases and GIS*

General Terms

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Keywords

shortest path, turn cost, near-shortest path

1. INTRODUCTION

Finding the fastest route on road networks has received a renewed interest in the recent past, thanks in large part to the proliferation of mobile location-aware devices. However, there exist many real-life scenarios in which the fastest route may not be the ideal choice when providing routing directions.

As a motivating example, consider the case of a tourist asking for driving directions to a specific landmark. Since the tourist may not be familiar with the neighborhood, it makes more sense to offer directions that involve as few turns as possible, instead of describing in detail an elaborate fastest route. As another example, consider an emergency situation, e.g., natural disaster, terrorist attack, which requires an evacuation plan to be communicated to people on the site. Under such circumstances of distress and disorganization, it is often desirable to provide concise, easy to memorize, and clear to follow instructions.

In both scenarios, the *simplest route* may be more preferable than the fastest route. As per the most common interpretation [22], turns (road changes) are assigned costs, and the simplest route is the one that has the lowest total turn cost, termed *complexity.* For simplicity, in the remainder of this work, we assume that all turns have equal cost equal to 1; the generalization to non-uniform costs is straightforward.

In some road networks, the simplest and the fastest route may be two completely different routes. Consider for example a large city, e.g., Paris, that has a large ring road encircling a dense system of streets. The simplest route between two nodes that lie on (or are close to) the ring, would be to follow the ring. On the other hand, the fastest route may involve traveling completely within the enclosing ring. As a result the length of the simplest route can be much larger than that of the fastest route, and vice versa.

Surprisingly, with the exception of [13], the trade-off between length and complexity in finding an optimal route has not received sufficient attention. Our work addresses this issue by studying the problem of finding routes that are as fast and as simple as possible.

In particular, we first study the *fastest simplest problem*, i.e., of finding the fastest among all simplest routes, which was the topic of [13]. We show that although, for this problem, a label-setting method (a variant of the basic Dijkstra's algorithm) cannot be directly applied on the road network, it is possible to devise a conceptual graph on which it can. In fact, our proposed algorithm is orders of magnitude faster than the baseline solution. Moreover, using a similar methodology, it is possible to efficiently solve the *simplest fastest problem.*

Subsequently, we investigate the length-complexity tradeoff and introduce two novel problems that relax the constraint that the returned routes must be either fastest or simplest. The *fastest near-simplest problem* is to find the fastest possible route whose complexity is not more than $1 + \epsilon$ times larger than that of the simplest route. On the other hand, the *simplest near-fastest problem* is to find the simplest possible route whose length is not more than $1+\epsilon$ times larger than that of the fastest route.

These near-optimal problems are significantly more difficult to solve compared to their optimal counterparts. The reason is that there cannot exist a principle of optimality, exactly because the requested routes are by definition *sub-optimal* in length and complexity. Therefore, one must exhaustively enumerate all routes, and only hope to devise pruning criteria to quickly discard unpromising sub-routes.

We propose two algorithms, based on route enumeration, for finding the simplest near-fastest route; their extension for the fastest near-simplest problem is straightforward. The first follows a depth-first search principle in enumerating paths, whereas the second is inspired by A* search. Both algorithms apply elaborate pruning criteria to eliminate from consideration a large number of sub-routes. Our experimental study shows that they run in less than 400 msec in networks of around 80,000 roads and 110,000 intesections.

The remainder of the paper is organized as follows. Section 2 formally defines the problems and reviews related work. Section 3 discusses the fastest simplest, and Section 4 the simplest near-fastest problem. Then, Section 5 presents our experimental study and Section 6 concludes the paper.

2. PRELIMINARIES

Section 2.1 presents the necessary definition, while Section 2.2 reviews relevant literature.

2.1 Definitions

Let V denote a set of nodes representing road intersections. A *road* r is a sequence of distinct nodes from V. Let R denote a set of roads, such that all nodes appear in at least one road, and any pair of consecutive nodes of some road do not appear in any other, i.e., the roads do not have overlapping subsequences. For a node $n \in V$, the notation $R(n) \subseteq R$ represents the non-empty subset of roads that contain n. For two consecutive nodes n_i, n_j of some road r, the notation $R(n_i, n_j)$ is a shorthand for r.

Definition 1. *The* road network *of R is the directed graph $G_R(V,E)$, where V is the set of nodes, and $E \subseteq V \times V$ contains an edge $e_{ij} = (n_i, n_j)$ if n_i, n_j are consecutive nodes in some road.*

A road network is associated with two cost functions. The *length function* L assigns to each edge a cost representing its length, i.e., the travel time or distance between them; formally, $L : E \to \mathbb{R}^+$ maps each edge (n_i, n_j) to the length $L(n_i, n_j)$ of the road segment n_i to n_j.

The *complexity function* C assigns to each turn from road r_i to road r_j via node n_x, which lies on both r_i and r_j, the cost of making the turn. Formally, $C : V \times R \times R \to \mathbb{R}^+$ maps (n_x, r_i, r_j) to complexity $C(n_x, r_i, r_j)$ from r_i to r_j via n_x.

A *route* $\rho = (n_a, n_b, \dots)$ is a path on graph G_R, i.e., a sequence of nodes from V, such that for any two consecutive nodes, say n_i, n_j, there exists an edge e_{ij} in E.

The *length* $L(\rho)$ of a route ρ is the sum of the lengths for each edge it contains, and represents the total travel time or distance covered along this route; formally,

$$L(\rho) = \sum_{(n_i,n_j)\in\rho} L(n_i, n_j). \tag{1}$$

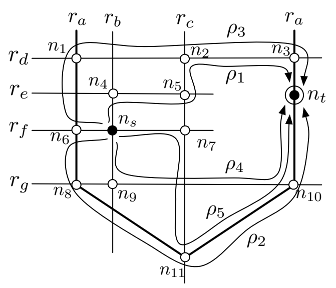


Figure 1: An example road network of seven roads $r_a - r_g$, where five routes $\rho_1 - \rho_5$ from node n_s to n_t are depicted.

Table 1: Costs of routes in Figure 1

road	length	complexity	type
ρ_1	10	4	SF
ρ_2	40	1	FS
ρ_3	20	3	SNF ($\epsilon = 1$)
ρ_4	30	2	FNS ($\epsilon = 1$)
ρ_5	40	2	—

A route from source n_s to target n_t is called a *fastest route* if its length is equal to the smallest length of any route from n_s to n_t. Given a parameter ϵ, a route from n_s to n_t is called an *near-fastest route* if its length is at most $(1+\epsilon)$ times that of the fastest route from n_s to n_t.

The *complexity* $C(\rho)$ of a route ρ is the sum of complexities for each turn it contains; formally

$$C(\rho) = \sum_{(n_i,n_j,n_k)\in\rho} C(n_j, R(n_i,n_j), R(n_j,n_k)), \tag{2}$$

where n_i, n_j, n_k are three consecutive nodes in ρ, and $R(n_i, n_j)$, $R(n_j, n_k)$ are the (unique) roads containing segments (n_i, n_j) and (n_j, n_k), respectively. A route from n_s to n_t is called a *simplest route* if its complexity is equal to the lowest complexity of any route from n_s to n_t. Given a parameter ϵ, a route from n_s to n_t is called a *near-simplest route* if its complexity is at most $(1+\epsilon)$ times that of the simplest route from n_s to n_t. Note that the complexity of a simplest route can be 0, i.e., when no road changes exist. In this case, all near-simplest routes must also have complexity 0. To address this, one could simply change the definition of complexity to be the number of roads in a route, and thus at least 1. In the remainder of this paper, we ignore this case, and simply use the original definition of complexity.

This work deals with the following problems. To the best of our knowledge only the first has been studied before in literature [13].

Problem 1. [**Fastest Simplest Route**] Given a source n_s and a target n_t, find a route that has the smallest length among all simplest routes from n_s to n_t.

Problem 2. [**Simplest Fastest Route**] Given a source n_s and a target n_t, find a route that has the smallest complexity among all fastest routes from n_s to n_t.

Problem 3. [**Fastest Near-Simplest Route**] Given a source n_s and a target n_t, find a route that has the smallest length among all near-simplest routes from n_s to n_t.

Problem 4. [**Simplest Near-Fastest Route**] Given a source n_s and a target n_t, find a route that has the lowest complexity among all near-fastest routes from n_s to n_t.

Note that the first two problems are equivalent to the last two, respectively, if we set $\epsilon = 0$. We next present an example illustrating these problems.

Example 1. Consider the road network of Figure 1 consisting of 7 two-way roads $r_a - r_g$. Note that all roads have either a north-south or an east-west direction, except road r_a, which is a ring-road and is thus depicted with a stronger line. The figure also portrays 11 road intersections n_1–n_{11} with hollow circles, and two special nodes, the source n_s, drawn with filled circle, and the target n_t, drawn with a filled circle inside a larger hollow one.

Next, consider five possible routes ρ_1–ρ_5 starting from n_s and ending at n_t, which are drawn in Figure 1, and whose lengths and complexities are shown in Table 1. Observe that ρ_1 is the fastest route from n_s to n_t with length 10, and, moreover, it has the lowest complexity 4 among all other fastest route (no other exists). Thus, ρ_1 is the simplest fastest route and the answer to Problem 2.

On the other hand, ρ_2 is the fastest simplest route and the answer to Problem 1, as it has the lowest complexity 1, following the ring-road to reach the target. But its length, 40, is quite large compared to the other possible routes.

Assume $\epsilon = 1$ for the complexity, so that a near-simplest route can have complexity at most twice that of the simplest route, i.e., 2. Observe that two routes ρ_4 and ρ_5 are near-simplest. Among them ρ is the fastest, and is thus the answer to Problem 3.

Moreover, assume $\epsilon = 1$ for the length as well, so that a near-fastest route can have length at most twice that of the fastest route, i.e., 20. Notice that only ρ_3 is near-fastest and thus is the answer to Problem 4.

Observe that if we set $\epsilon = 2$ for the length, near-fastest routes can have length as large as 30. In this case, ρ_3 and ρ_4 are near- fastest, with the latter being the simplest near-fastest. □

2.2 Related Work

Dijkstra [7] showed that the fastest route problem exhibits a principle of sub-route optimality and proposed its famous dynamic programming method for finding all fastest routes from a given source. Bi-directional search [17], i.e., initiating two parallel searches from the source and the target can significantly expedite finding the fastest source-to-target route. Since this early work around the 60's, numerous network preprocessing techniques exist today, including landmarks [11], reach [12], multi-level graphs [21], graph hierarchies [20, 9], graph partitioning [16], labelings [1], and their combinations [2, 3, 6], which are capable of speeding up Dijkstra's algorithm by orders of magnitude in several instances.

The problem of finding the simplest route was first studied in [5], and more recently in [22, 8]. The basic idea behind these methods, is to construct a pseudo-dual graph of the road network, where road segments become the nodes, the turns between two consecutive road segments become the edges, which are assigned turn costs. Then, finding the simplest route reduces to finding the shortest path on the transformed graph. In contrast, the recent work of [10] solves the simplest route problem directly on the road network. Note that Problem 1 differs with respect to the simplest route problem, as it request a specific simplest route, that with the smallest length. The aforementioned methods return any simplest route.

To the best of our knowledge, only the work in [13] addresses Problem 1, as it proposes a solution that first finds all simplest routes and then selects the fastest among them. The proposed method serves as a baseline approach to our solution for Problem 1 and is detailed in Section 3.1.

Problems 3 and 4 are related to the problem of finding the near-shortest paths on graphs [4, 15]. This paper differs with that line of work in two ways. First, the studied problems involve two cost metrics, length and complexity. Second, their solution is a single route, instead of all possible near-optimal routes.

Problems 3 and 4 are also related to multi-objective shortest path problems (see e.g., [18, 14]), which specify more than one criteria and may return sub-optimal routes. This paper differs with that line of work again in two ways. First, they request a single route. Second, the studied problems introduce a hard constraint on the length or complexity of a solution. Nonetheless, an interesting extension to our work would be to return all routes that capture different length-complexity trade-offs.

3. FASTEST SIMPLEST ROUTE

This section discusses Problem 1, and introduces an algorithm that takes advantage of the principle of sub-route optimality to expedite the search. Solving Problem 2 is similar, and thus details are omitted. We first present a recent baseline solution in Section 3.1, and then discuss our approach in Section 3.2.

3.1 Baseline Solution

The work in [13] was the first to address the fact that there can exist multiple simplest routes with greatly varying length, and proposes a solution to finding the fastest among them. This method, which we denote as BSL, operates on a graph that models the *intersections* of the roads in R.

Definition 2. *The* intersection graph *of R is the undirected graph $G_I(R, I)$, where R is the set of roads; and $I \subseteq R \times R \times V$ contains intersection (n_x, r_i, r_j) if $r_i \in R(n_x)$ and $r_j \in R(n_x)$, i.e., node n_x belongs to both roads r_i and r_j.*

A path on the intersection graph, i.e., a sequence of R vertices such that there exists an intersection in I for any two consecutive vertices in the sequence, is called a *road sequence.*

BSL finds the fastest simplest route from a source node n_s to a target node n_t. It is based on the observation that a simplest route from n_s to n_t in the road network is related to a shortest road sequence from a road that contains n_s to one that contains n_t in the intersection graph. More precisely, BSL operates as follows.

1. For each source road in $R(n_s)$, find the number of intersections of the shortest road sequence from that source to any of the target roads in $R(n_t)$, e.g., using a single-source shortest path algorithm on the intersection graph.
2. Determine the smallest number of intersections among those found in the previous step. This number corresponds to the fewest possible intersections in a road sequence that starts from a source and ends at a target road, and is thus equal to the complexity of the simplest route from n_s to n_t plus 1.
3. Enumerate (e.g., using depth-limited dfs) all road sequences from a source to a target road that have exactly

as many intersections as the number determined in the previous step. For each road sequence produced, convert it to a route and determine its length.

4. Select the route with the minimum length, i.e., the fastest, among those produced in the previous step.

3.2 The FastestSimplest Algorithm

The proposed algorithm operates directly on the road network. However, a direct application of a Dijkstra-like (label-setting [7]) method is not possible, because the principle of sub-route optimality does not hold. In particular, this principle suggests that if node n_x is in the fastest simplest route from n_s to n_t then *any* fastest simplest sub-route from n_s to n_x can be extended to a fastest simplest route from n_x to n_t. In comparison, it is easy to see that the principle holds for fastest route (shortest paths), as *all* fastest sub-routes can be extended to fastest routes.

We give a counter-example for the principle of optimality on fastest simplest routes using the road network of Figure 1. Consider the routes $\rho_2 = (n_s, n_6, n_8, n_{11}, n_{10}, n_t)$ and $\rho_5 = (n_s, n_7, n_{11}, n_{10}, n_t)$. The sub-route $\rho_5' = (n_s, n_7, n_{11})$ of ρ_5 has length 20 and complexity 1, as it involves a single turn from road r_f to r_c via node n_7. Similarly, the sub-route $\rho_2' = (n_s, n_6, n_8, n_{11})$ of ρ_2 has length 20 and complexity 1, as it involves a single turn from r_f to r_a via node n_6. Therefore, both sub-routes are fastest simplest from n_s to n_{11}. However, the extension of ρ_5' does not give a fastest simplest route from n_s to n_t; ρ_5 makes an additional turn at node n_{11} compared to ρ_2. This violates the principle of optimality for fastest simplest routes.

Therefore, a Dijkstra-like method, which directly exploits this principle of optimality, cannot be applied. For instance, such a method could reach n_{11} first via r_c and subsequently ignore any other path reaching n_{11}, including the sub-route via r_a, and thus missing the optimal route from n_s to n_t.

To address the aforementioned lack of sub-route optimality, we construct a conceptual expanded graph, on which the principle optimality holds. Additionally, we show that expanded routes on this graph are uniquely associated with routes on the road network. We emphasize that the expanded graph is only a conceptual structure used for presentation purposes, and that the proposed algorithm does not make use of it as it operates directly on the road network.

Definition 3. *The* expanded graph *of $G_R(V, E)$ is the directed graph $G_\mathcal{E}(V', E')$, where $V' \subseteq V \times R$ contains an expanded node (n_x, r_i) if $r_i \in R(n_x)$; $E' \subseteq V' \times V'$ contains an edge $((n_x, r_i), (n_y, r_j))$ if $r_i \in R(n_x)$ and $r_j \in R(n_x)$ (r_i and r_j could be the same road), and additionally n_x, n_y are consecutive nodes in r_j.*

An expanded route $\rho_\mathcal{E} = ((n_a, r_i), (n_b, r_j), \dots)$ is a path on the expanded graph $G_\mathcal{E}$. Each expanded edge can be associated with a length and a turn cost. Therefore, it is possible to define the following costs for an expanded route.

The length $L(\rho_\mathcal{E})$ of an expanded route $\rho_\mathcal{E}$ is the sum of the lengths associated with each expanded edge; formally,

$$L(\rho_\mathcal{E}) = \sum_{((n_x,r_i),(n_y,r_j)) \in \rho_\mathcal{E}} L(n_x, n_y). \tag{3}$$

Similarly, the complexity $C(\rho_\mathcal{E})$ of an expanded route $\rho_\mathcal{E}$ is the sum of the turn costs associated with each expanded edge; formally,

$$C(\rho_\mathcal{E}) = \sum_{((n_x,r_i),(n_y,r_j)) \in \rho_\mathcal{E}} C(n_x, r_i, r_j). \tag{4}$$

An important property regarding the length and complexity of an expanded route is the following. Note that a similar propery does not generally hold for routes on the road network G_R.

Lemma 1. *Let $\rho_\mathcal{E}^1$ be an expanded route from (n_s, r_i) to (n_x, r_y), and $\rho_\mathcal{E}^2$ be an expanded route from (n_x, r_y) to (n_t, r_j). If $\rho_\mathcal{E}^1 \rho_\mathcal{E}^2$ denotes the concatenation of the two expanded routes, then, it holds that $L(\rho_\mathcal{E}^1 \rho_\mathcal{E}^2) = L(\rho_\mathcal{E}^1) + L(\rho_\mathcal{E}^2)$, and $C(\rho_\mathcal{E}^1 \rho_\mathcal{E}^2) = C(\rho_\mathcal{E}^1) + C(\rho_\mathcal{E}^2)$.*

Please note that the proofs of all lemmas and theorems can be found in the technical report [19].

It should be apparent that expanded routes are closely related with (non-expanded) routes. First, let us examine the G_R to $G_\mathcal{E}$ relationship, which is one to many.

We associate a route ρ from n_s to n_t on the road network G_R to a set $\mathcal{E}(\rho)$ of expanded routes on $G_\mathcal{E}$, which only differ in their first and last expanded nodes. Particularly, for an expanded route $\rho_\mathcal{E} \in \mathcal{E}(\rho)$, its first expanded node is (n_s, r_i), where $r_i \in R(n_s)$, the last expanded node is (n_t, r_j), where $r_j \in R(n_t)$, and the k-th expanded node (for $k > 1$) is $(n_k, R(n_{k-1}, n_k))$, where n_{k-1}, n_k are the $(k-1)$-th, k-th nodes in ρ, respectively, and $R(n_{k-1}, n_k)$ is the unique road that contains the edge (n_{k-1}, n_k). Conversely, an expanded route $\rho_\mathcal{E}$ is associated with a unique route ρ.

Given a route ρ, we define the *special expanded route* of ρ, denoted as $\rho_\mathcal{E}^*$, to be the expanded route in $\mathcal{E}(\rho)$ that has $(n_s, R(n_s, n_{s+1}))$ as its first expanded node, and $(n_t, R(n_{t-1}, n_t))$ as its last expanded node, where n_{s+1} is the second node in route ρ, $R(n_s, n_{s+1})$ is the unique road containing edge (n_s, n_{s+1}), n_{t-1} is the second-to-last node in route ρ, and $R(n_{t-1}, n_t)$ is the unique road containing edge (n_{t-1}, n_t).

An even more important property is the following.

Lemma 2. *The length of a route ρ is equal to the length of any expanded route $\rho_\mathcal{E} \in \mathcal{E}(\rho)$. The complexity of a route ρ is equal to the complexity of the special expanded route $\rho_\mathcal{E}^* \in \mathcal{E}(\rho)$.*

Next, let us examine the $G_\mathcal{E}$ to G_R relationship, which is many to one. We associate an expanded route $\rho_\mathcal{E}$ from (n_s, r_i) to (n_t, r_j) to a unique route $\rho = \mathcal{E}^{-1}(\rho_\mathcal{E})$ from n_s to n_t on G_R, such that the k-th node (for any k) of ρ is n_k, where (n_k, r_x) is the k-th expanded node of $\rho_\mathcal{E}$.

Lemma 3. *The length of an expanded route $\rho_\mathcal{E}$ is equal to the length of the route $\rho = \mathcal{E}^{-1}(\rho_\mathcal{E})$. The complexity of a route $\rho_\mathcal{E}$ is not smaller than the complexity of the route $\rho = \mathcal{E}^{-1}(\rho_\mathcal{E})$.*

We next introduce a lexicographic total order, which applies to routes or expanded routes. Note that in this section, we use this order exclusively for expanded routes. Given two routes ρ^1, ρ^2, we say that ρ^1 is FS-shorter than ρ^2 and denote as $\rho^1 <_{FS} \rho^2$ if $C(\rho^1) < C(\rho^1)$ or if $C(\rho^1) = C(\rho^1)$ and $L(\rho^1) < L(\rho^1)$. Intuitively, being FS-shorter implies being simpler or as simple but faster.

The following theorem presents an important property regarding this order on expanded routes.

Theorem 1. *Let ρ^{FS} be a fastest simplest route on G_R from n_s to n_t, and let $\rho_{\mathcal{E}}^{FS*}$ denote its special expanded route. It holds that there exists no other expanded route that starts from (n_s, r_i) and ends at (n_t, r_j), for any $r_i \in R(n_s)$ and $r_j \in R(n_t)$ that is FS-shorter than $\rho_{\mathcal{E}}^{FS*}$.*

Theorem 1 implies that to find a fastest simplest route on G_R, it suffices to find a FS-shortest expanded route on $G_{\mathcal{E}}$.

The following theorem shows that a principle of optimality holds for FS-shortest expanded routes on $G_{\mathcal{E}}$.

Theorem 2. *Let $\rho_{\mathcal{E}}$ denote an FS-shortest expanded route from (n_s, r_i) to (n_t, r_j) that passes through (n_x, r_y). Furthermore, let $\rho_{\mathcal{E}}^1$ denote its sub-route from (n_s, r_i) to (n_x, r_y), and $\rho_{\mathcal{E}}^2$ its sub-route from (n_x, r_y) to (n_t, r_j). It holds that both $\rho_{\mathcal{E}}^1$ and $\rho_{\mathcal{E}}^2$ are FS-shortest. Moreover, if $\rho_{\mathcal{E}}^{1'}$ is another FS-shortest expanded route from (n_s, r_i) to (n_x, r_y), then $\rho_{\mathcal{E}}^{1'} \rho_{\mathcal{E}}^2$ is an FS-shortest expanded route from (n_s, r_i) to (n_t, r_j).*

The key point in Theorem 2 is that it holds for *any* expanded route on $G_{\mathcal{E}}$. In contrast, this does not hold for routes on the road network G_R, as we have argued in the beginning of this section.

Given Theorem 2, we can apply a Dijkstra's algorithm, or any variant, to find the FS-shortest expanded route on $G_{\mathcal{E}}$. Then, from Theorem 1, we immediately obtain a fastest simplest route on G_R.

In what follows, we present the FastestSimplest (FS) algorithm, a label- setting method (a generalization of Dijkstra's algorithm) for finding a fastest simplest route on G_R, which operates directly on the road network and constructs directly routes, instead of expanded routes.

The pseudocode of the FS algorithm is depicted in Algorithm 1. Although FS operates on the road network G_R, it updates labels for expanded nodes. A label $\lambda(n, r)$ for expanded node (n, r) is equal to $\langle n, r | len, cpl, n_{prev}, r_{prev} \rangle$ and represents an expanded route from (n_s, r_i), for some $r_i \in R(n_s)$, up to (n, r). In particular, *len*, *cpl* are the length and complexity of this expanded route, while (n_{prev}, r_{prev}) is the second-to-last expanded node. Note that this expanded route is FS-shortest only when it is explicitly marked as `final`.

FS uses a minheap H to guide the search, visiting nodes of G_R. An entry of H is a label, and its key is the label's length, complexity pair (len, cpl). Labels in H are ordered using the FS-shorter total order. At each iteration, FS deheaps a label, marks it `final` and advances the search frontier.

For any road $r_i \in R(n_s)$, the algorithm initializes the heap with the label $(n_s, r_i | 0, 0, n_\varnothing, r_i)$ (lines 1–3). The dummy node $n_\varnothing$ signifies that n_s is the first node in any route constructed.

The algorithm proceeds iteratively, deheaping labels until the heap is depleted (line 4), or the label involving the target is deheaped (line 7). Assume $(n_x, r_i | len, cpl, n_w, r_h)$ is the deheaped label (line 5). As explained before, this label is finalized (line 6).

If the label does not involve the target, FS expands the current route (represented by the deheaped label) considering each outgoing edge (n_x, n_y) of n_x (line 8), and each road r_j that contains n_y (line 9).

If the label $\lambda(n_y, r_j)$ does not exist (line 10), its label is initialized with length equal to *len* plus the distance $L(n_x, n_y)$ of the outgoing edge, and with complexity equal to *cpl* plus

Algorithm 1: FastestSimplest

```
Input: road network $G_R$; function $L$; function $C$; source $n_s$;
       target $n_t$
Output: length $fsL$ and complexity $fsC$ of fastest simplest
        route from $n_s$ to $n_t$
Variables: minheap $H$ with entries $\langle n, r | len, cpl, n_{prev}, r_{prev} \rangle$,
           keys $(len, cpl)$, and compare function $<_{FS}$
1  foreach $r_i$ that contains $n_s$ do
2      $\lambda(n_s, r_i) \leftarrow \langle n_s, r_i | 0, 0, n_\varnothing, r_i \rangle$
3      enheap $\lambda(n_s, r_i)$ in $H$
4  while $H$ not empty do
5      $\langle n_x, r_i | len, cpl, n_w, r_h \rangle \leftarrow$ deheap
6      mark $\lambda(n_x, r_i)$ as final
7      if $n_x$ is $n_t$ then break
8      else foreach edge $(n_x, n_y)$ do
9          foreach road $r_j$ that contains $n_y$ do
10             if $\lambda(n_y, r_j)$ does not exist then
11                 $\lambda(n_y, r_j) \leftarrow$
                   $\langle n_y, r_j | len + L(n_x, n_y), cpl + C(n_y, r_i, r_j), n_x, r_i \rangle$
12                 enheap $\lambda(n_y, r_j)$
13             else if $\lambda(n_y, r_j)$ is not final then
14                 $\langle n_y, r_j | len', cpl', n_u, r_h \rangle \leftarrow \lambda(n_y, r_j)$
15                 if
                   $(len + L(n_x, n_y), cpl + C(n_y, r_i, r_j)) <_{FS} (len', cpl')$
                   then
16                     $\lambda(n_y, r_j) \leftarrow$
                       $\langle n_y, r_j | len + L(n_x, n_y), cpl + C(n_y, r_i, r_j), n_x, r_i \rangle$
17                     update $\lambda(n_y, r_j)$
18
19 return $(fsL, fsC) \leftarrow (len, cpl)$
```

the complexity $C(n_y, r_i, r_j)$ of transitioning from road r_i to r_j via node n_y (lines 11 –12).

Otherwise, if label $\lambda(n_y, r_j)$ exists but is not `final` (line 13), it is retrieved (line 14). The label will be updated if the extension of the current expanded route is FS-shorter that the one currently represented in the label (lines 15–17).

The fastest simplest route can be retrieved with standard backtracking. We keep all deheaped labels, and then starting from the label containing the target, we identify the previous expanded node (from the information stored in the label) and retrieve its label, until the source is reached.

Theorem 3. *The FS algorithm correctly finds a fastest simplest route from n_s to n_t.*

Analysis. Let $\delta = \max_{n \in V} |R(n)|$ denote the maximum degree of the road network G_R, i.e., the maximum number of roads a node can belong to. Note that there exist not more than $\delta|V|$ labels, i.e., (n, r) pairs. In the worst case, FastestSimplest performs an enheap and deheap operation for each label. Furthermore, in the worst case, FastestSimplest examines each edge δ times, one for each label of a node. For each examination, it may update δ labels, in the worst case. Therefore, there is a total of $\delta^2|E|$ updates, in the worst case. Assuming a Fibonacci heap, the time complexity of FastestSimplest is $O(\delta^2|E| + \delta|V| \log |V|)$ amortized. Moreover, since the heap may contain an entry for each label, the space complexity is $O(\delta|V|)$.

Discussion. Thanks to Theorem 2, the FS algorithm essentially solves a shortest path problem defined on the expanded graph directly on the road network. It is thus possible to substitute the underlying basic label-setting method method with a more efficient variant. Bi- directional search and all graph preprocessing techniques, discussed in Section 2.2, are compatible and can expedite the underlying method.

4. SIMPLEST NEAR-FASTEST ROUTE

This section studies Problem 4; the solution to Problem 2 is similar and details are omitted. Unlike the case of finding the simplest fastest or the fastest simplest route, there can exist no principle of optimality, exactly because the solution to Problem 4 is *not an optimal route* for any definition of optimality. Therefore, one has to enumerate all routes from source to target, and rely on bounds and pruning criteria to eliminate sub-routes that cannot be extended to simplest near-fastest route.

We propose two algorithms, which differ in the way they enumerate paths. The first, detailed in Section 4.1, is based on depth-first search, while the second, detailed in Section 4.2, is inspired by A* search.

4.1 DFS-based Traversal

This section details the SimplestNearFastest-DFS (SNF-DFS) algorithm for finding the simplest near-fastest route. Its key idea is to enumerate all routes from source to target by performing a depth-first search, eliminating in the process routes which are longer than $(1+\epsilon)$ times the fastest (similar to the algorithm of [4] for near-fastest routes), or have larger complexity than the best found so far.

SNF-DFS requires information about the simplest fastest as well as the fastest simplest path from any node to the target. To obtain this information, it invokes two procedures AllFastestSimplest and AllSimplestFastest.

The AllFastestSimplest procedure is a variation of the FastestSimplest algorithm (Section 3) that solves the single-source fastest simplest route problem, i.e., it computes the length and complexity of the fastest simplest route from a given source to any other node. Only a small change to the original algorithm is necessary. Recall that when deheaping a label $\lambda(n_x, r_i)$, it is marked as final. Observe that when the first label associated with n_x is deheaped, the algorithm has found the fastest simplest path from n_s to n_x. (This was in fact the termination condition of Algorithm 1: stop when a label associated with the target is deheaped.) Therefore, the AllFastestSimplest procedure explicitly marks n_x as visited at its first encounter, and stores the length and complexity of the current path. The procedure only terminates when the heap empties.

The AllSimplestFastest procedure is derived from the SimplestFastest algorithm in the same way that AllFastestSimplest is from FastestSimplest, and thus details are omitted.

Note that there arises a small implementation detail. Recall that the SNF-DFS algorithm requires the costs all fastest simplest routes *ending at* a particular node (the target), whereas AllFastestSimplest returns the costs of all fastest simplest routes *starting from* a particular node. Therefore, to obtain the appropriate info, SNF-DFS invokes the AllFastestSimplest procedure using a graph obtained from G_R by inverting the direction of its edges. The same holds for the invokation of the AllSimplestFastest procedure.

In the following, we assume that the length $fsL[\,]$ and complexity $fsC[\,]$ of all fastest simplest routes to the target n_t, and the length $sfL[\,]$ and complexity $sfC[\,]$ of all simplest fastest routes to n_t, are given.

The SNF-DFS algorithm applies two pruning criteria to avoid examining all routes from n_s to n_t.

Lemma 4. *Let ρ be a route from n_s to n_x. If $L(\rho) + sfL[n_x] > (1+\epsilon) \cdot sfL[n_s]$, then any extension of ρ towards n_t is not a simplest near-fastest route.*

Lemma 5. *Let ρ be a route from n_s to n_x. Further, let $snfC^+$ be an upper bound on the complexity of a simplest near-fastest route from n_s to n_t. If $C(\rho) + fsC[n_x] > snfC^+$, then any extension of ρ towards n_t is not a simplest near-fastest route.*

The next lemma computes an upper bound of the complexity of a simplest near-fastest route.

Lemma 6. *Let ρ be a route from n_s to n_x. If $L(\rho) + fsL[n_x] \leq (1+\epsilon) \cdot sfL[n_s]$, then $snfC^+ = C(\rho) + 1 + fsC[n_x]$ is an upper bound on the complexity of a simplest near-fastest route.*

We are now ready to describe in detail the SNF-DFS algorithm, whose pseudocode is shown in Algorithm 2. It performs a depth-first search on the road network, eliminating routes according to the two criteria described previously, and computing an upper bound for the complexity of the fastest near-simplest route.

SNF-DFS uses a stack S to implement depth-first search. At each point in time, the entries in the stack S form exactly a single route starting from n_s. An entry of S has the form $\langle n | len, cpl, n_{prev} \rangle$, and corresponds to a route ending at node n with length len, complexity cpl and whose second-to-last node is n_{prev}.

SNF-DFS marks certain nodes as `in_route`, and certain edges as `traversed`. Particularly, a node is marked as `in_route` if an entry for this node is currently in the stack. This marking helps avoid cycles in routes. An edge (n_a, n_b) is marked as `traversed` if an entry for n_a is in the stack (not necessarily the last), whereas an entry for n_b is not in S, but was at some previous iteration right above the entry for n_a. This marking helps avoid revisiting routes.

Initially, SNF-DFS invokes the AllSimplestFastest and AllFastestSimplest procedures (lines 1–2). Then, if the fastest simplest route from n_s to n_t is near-shortest, i.e., has length less than $(1+\epsilon) \cdot sfL[n_s]$, then it is not only a candidate route but actually the solution, as there can be no other route with lowest complexity. Hence, SNF-DFS terminates (lines 3–4).

Otherwise, a candidate route is the fastest simplest route, which is definitely near-fastest. Therefore, an upper bound on complexity is computed as $snfC^+ = sfC[n_s]$ (line 5). The stack is initialized with an entry for the source node n_s (line 6). Then SNF-DFS proceeds iteratively until the stack is empty (line 7).

At each iteration the top entry of the stack is examined (but not popped) (line 8). Let this entry be for node n_x and correspond to a route ρ. If node n_x is not marked as `in_route` although it is at the top of the stack, this means that this is the first time SNF-DFS encounters it (line 9). For this first encounter, the algorithm applies the pruning criterion of Lemma 5. If it holds (line 10) then no route that extends ρ will be examined, and hence the entry is popped from the stack.

Otherwise, if n_x is the target (lines 11–13), ρ constitutes a candidate solution and its complexity is compared against the best known (line 12). Subsequently, the entry is popped, as there is no need to extend the current route ρ any farther. If the entry is not popped, node n_x is marked as `in_route`.

If node n_x is not `in_route`, SNF-DFS looks for an outgoing edge (n_x, n_y) such that it is not `traversed` and n_y is not `in_route` (line 16). If no such edge is found, then all routes, with no cycles, that extend ρ have been either considered or pruned. Hence the top entry of the stack is popped (line 18), and n_x is marked as not `in_route` (line 19). Additionally, all outgoind edges of n_x are marked as not `traversed` (lines 20–21).

Otherwise, such an outgoing edge (n_x, n_y) is found. Then, the algorithm checks if the two pruning criteria (Lemmas 4, 5) apply (line 22). If either does, then the edge (n_x, n_y) is marked as `traversed` (line 27). Otherwise (lines 23– 26), the algorithm checks if Lemma 6 applies, and appropriately updates the complexity bound $snfC^+$ if necessary (line 24). Finally, SNF-DFS creates an entry for node n_y and pushes it in the stack (line 25), while marking (n_x, n_y) as `traversed` (line 26).

The actual simplest near-fastest route can be retrieved with standard backtracking; details are omitted.

Theorem 4. *The SNF-DFS algorithm correctly finds a simplest near-fastest route from n_s to n_t.*

Analysis. The complexities of AllSimplestFastest and AllFastestSimplest are the same as those of SimplestFastest and FastestSimplest, respectively, namely $O(\delta^2|E|+\delta|V|\log|V|)$ amortized time and $O(\delta|V|)$ space.

Let $L(\rho^{SF})$ denote the length of the fastest route, and Δd the smallest distance of any edge. At any time the stack of SimplestNearFastest corresponds to a sub-route of some near-fastest route. The number of edges in a near-fastest route can be at most $(1+\epsilon)L(\rho^{SF})/\Delta d$ (but not more than $|E|$). Therefore, the space complexity of the road network traversal is $O((1+\epsilon)L(\rho^{SF})/\Delta d) = O(|E|)$, since at each time a single route is maintained in the stack.

In the worst case, the traversal may examine all possible routes from n_s to n_t having $(1+\epsilon)L(\rho^{SF})/\Delta d$ edges. The number of such routes is $k = \binom{|E|}{(1+\epsilon)L(\rho^{SF})/\Delta d}$; in practice this is a much smaller number. The number of push or pop operations is in the worst case equal to the total length of all possible near-fastest routes from source to target. Since there can be k such routes, the time complexity is $O(k(1+\epsilon)L(\rho^{SF})/\Delta d)$.

Overall, the time complexity of SNF-DFS is $O(\delta^2|E| + \delta|V|\log|V| + k(1+\epsilon)L(\rho^{SF})/\Delta d)$ amortized, while its space complexity is $O(\delta|V| + (1+\epsilon)L(\rho^{SF})/\Delta d)$.

Discussion. The running time of SNF-DFS depends on large part on the two procedures AllSimplestFastest and AllFastestSimplest. In the following, we discuss a variant of the algorithm that does not invoke these procedures. The key idea is to relax the requirement for explicit calculation of the length and complexity of all simplest fastest and fastest simplest routes, and instead require a method for calculating their lower and upper bounds. Such a method can be straightforwardly adapted from landmark-based techniques, e.g., [11]. Note that in the extreme case, no bounds are necessary. Clearly, the pruning criteria of Lemmas 4 and 5 can be straightforwardly adapted to use bounds instead; note that their pruning power is reduced. Similarly, Lemma 6 can also be adapted, which results however in a less tight upper bound. Details are omitted.

4.2 A*-based Traversal

This section describes the SimplestNearFastest-A* (SNF-A*) algorithm for finding a simplest near-fastest route, which is inspired by A* search. The key idea is to use bounds on the complexity in order to guide the search towards the simplest among the near-fastest routes.

Algorithm 2: SimplestNearFastest-DFS

```
Input: road network $G_R$; mapping $C$; source $n_s$; target $n_t$;
       value $\epsilon$
Output: length $snfL$ and complexity $snfC$ of simplest
        near-fastest route from $n_s$ to $n_t$
Variables: stack $S$ with entries $\langle n|len, cpl, n_{prev}\rangle$
1  $(sfL[\,], sfC[\,]) \leftarrow$ AllSimplestFastest$(G_R, C, n_t)$
2  $(fsL[\,], fsC[\,]) \leftarrow$ AllFastestSimplest$(G_R, C, n_t)$
3  if $fsL[n_s] \leq (1+\epsilon)\cdot sfL[n_s]$ then
4      return $(snfL, snfC) \leftarrow (fsL[n_s], fsC[n_s])$
5  $(snfL^+, snfC^+) \leftarrow (sfL[n_s], sfC[n_s])$
6  push $(n_s|0, 0, n_\varnothing)$
7  while $S$ not empty do
8      $\langle n_x|len, cpl, n_w\rangle \leftarrow$ top
9      if $n_x$ not in_route then
10         if $cpl + fsC[n_x] > snfC^+$ then pop
11         else if $n_x$ is $n_t$ then
12             if $cpl < snfC^+$ then $(snfL^+, snfC^+) \leftarrow (len, cpl)$
13             pop
14         else mark $n_x$ as in_route
15     else
16         find an outgoing edge $(n_x, n_y)$ that is not traversed and
           $n_y$ is not in_route
17         if no such edge is found then
18             pop
19             mark $n_x$ as not in_route
20             foreach outgoing edge $(n_x, n_y)$ do
21                 mark $(n_x, n_y)$ as not traversed
22         else if $len + L(n_x, n_y) + sfL[n_y] \leq (1+\epsilon)\cdot sfL[n_s]$ and
           $cpl + C(e_{wx}, e_{xy}, n_x) + fsC[n_y] < snfC^+$ then
23             if $len + L(n_x, n_y) + fsL[n_y] \leq (1+\epsilon)\cdot sfL[n_s]$ then
24                 $(snfL^+, snfC^+) \leftarrow (len + L(n_x, n_y) + fsL[n_y], cpl +$
                   $C(e_{wx}, e_{xy}, n_x) + 1 + fsC[n_y])$
25             push $\langle n_y|len + L(n_x, n_y), cpl + C(e_{wx}, e_{xy}, n_x), n_x\rangle$
26             mark $(n_x, n_y)$ as traversed
27         else mark $(n_x, n_y)$ as traversed
28
29 return $(snfL, snfC) \leftarrow (snfL^+, snfC^+)$
```

Similar to the dfs-like algorithm, SNF-A* applies Lemmas 4, 5 to prune unpromising routes, and Lemma 6 to compute an upper bound on the complexity of a simplest near-fastest route. On the other hand, contrary to the dfs-like algorithm, SNF-A* terminates when it enounters the target node for the first time, because it can guarantee that all unexamined routes have more complexity.

The SNF-A* algorithm uses a heap to guide the search, containing node labels. An important difference with respect to the methods of Section 3, is that to guarantee correctness, there may be multiple labels per node, each corresponding to different routes from the source to that node. The reason is that there is no principle of optimality for near-fastest routes. Still, labels belonging to certain routes can be eliminated, as the following lemma suggests.

Lemma 7. *Let ρ, ρ' be two routes from n_s to n_x. If $L(\rho') > L(\rho)$ and $C(\rho') > C(\rho)+1$, then ρ' cannot be a sub-route of a simplest near-fastest route from n_s to any n_t.*

The set of labels for a node n_x is denoted by $\Lambda(n_x)$. Let λ (resp. λ') be the label corresponding to a route ρ (resp. ρ') ending at node n_x. If the conditions of Lemma 7 hold for ρ and ρ', we write $\lambda \prec \lambda'$. Clearly, there is no need to keep a label $\lambda' \in \Lambda(n_x)$ if there is another label $\lambda \in \Lambda(n_x)$ such that $\lambda \prec \lambda'$.

An important difference to the label-setting method for Problem 1 is that a heap entry (label) $\langle n|len, cpl, n_{prev}\rangle$ in SNF-A* is sorted according to the FS-shorter total order (see

Algorithm 3: SimplestNearFastest-A*

```
   Input: road network $G_R$; mapping $C$; source $n_s$; target $n_t$;
          value $\epsilon$
   Output: length $snfL$ and complexity $snfC$ of simplest
           near-fastest route from $n_s$ to $n_t$
   Variables: minheap $H$ with entries $\langle n|len, cpl, n_{prev}\rangle$, keys
              $(len + fsL[n], cpl + fsC[n])$, compare function $<_{FS}$
1  $(sfL[\,], sfC[\,]) \leftarrow$ AllSimplestFastest$(G_R, C, n_t)$
2  $(fsL[\,], fsC[\,]) \leftarrow$ AllFastestSimplest$(G_R, C, n_t)$
3  if $fsL[n_s] \leq (1+\epsilon) \cdot sfL[n_s]$ then
4      return $(snfL, snfC) \leftarrow (fsL[n_s], fsC[n_s])$
5  $(snfL^+, snfC^+) \leftarrow (sfL[n_s], sfC[n_s])$
6  enheap $\langle n_s|0, 0, n_\varnothing\rangle$ in $H$
7  while $H$ not empty do
8      $\langle n_x|len, cpl, n_w\rangle \leftarrow$ deheap
9      if $n_x$ is $n_t$ then
10         $(snfL^+, snfC^+) \leftarrow (len, cpl)$
11         break
12     else foreach edge $(n_x, n_y)$ do
13         $\lambda \leftarrow \langle n_y|len + L(n_x, n_y), cpl + C(e_{wx}, e_{xy}, n_x), n_x\rangle$
14         $pruned \leftarrow$ false
15         foreach entry $\lambda' \in \Lambda(n_y)$ do
16             if $\lambda \prec \lambda'$ then remove $\lambda'$
17             else if $\lambda' \prec \lambda$ then $pruned \leftarrow$ true
18         if not $pruned$ and
           $len + L(n_x, n_y) + sfL[n_y] \leq (1+\epsilon) \cdot sfL[n_s]$ and
           $cpl + C(e_{wx}, e_{xy}, n_x) + fsC[n_y] < snfC^+$ then
19             if $len + L(n_x, n_y) + fsL[n_y] \leq (1+\epsilon) \cdot sfL[n_s]$ then
20                 $(snfL^+, snfC^+) \leftarrow (len + L(n_x, n_y) + fsL[n_y], cpl +$
                   $C(e_{wx}, e_{xy}, n_x) + 1 + fsC[n_y])$
21             enheap $\lambda$
22
23 return $(snfL, snfC) \leftarrow (snfL^+, snfC^+)$
```

Section 3) on pair $(len + fsL[n], cpl + fsC[n])$, as it would in A* search.

The pseudocode of SNF-A* is shown in Algorithm 3. Initially, it invokes the AllSimplestFastest and AllFastestSimplest procedures to obtain arrays $fsL[\,]$, $fsC[\,]$, $sfL[\,]$, and $sfC[\,]$ (lines 1–2). Subsequently, if the fastest simplest route from n_s to n_t is near-shortest, it is the solution, and hence, SNF-DFS terminates (lines 3–4). Otherwise, a candidate route is the fastest simplest route, which is definitely near-fastest. Therefore, an upper bound on complexity is computed as $snfC^+ = sfC[n_s]$ (line 5).

The heap is initialized with an entry for the source node n_s (line 6). Then SNF-A* proceeds iteratively until the heap is empty (line 7). Let $\langle n_x|len, cpl, n_w\rangle$ be the deheaped label at some iteration (line 8). If n_x is the target, the algorithm terminates (lines 9–11). The reason is that because of the order in the heap, all remaining labels correspond to routes, which when extended via the simplest route to the target, have larger complexity. Hence, Lemma 5 applies to them.

If n_x is not the target, each outgoing edge (n_x, n_y) is examined (line 12), and a label λ for the route to n_y is created (line 13). Subsequently, each other label λ' regarding node n_y is considered (lines 15–17). In particular, the algorithm applies Lemma 7 for the routes of labels λ and λ', removing labels if necessary.

If the route for label λ survives, then the pruning criteria of Lemmas 4 and 5 are applied (line 18). If the label still survives, then Lemma 6 is applied to compute an upper bound on the complexity of a solution (lines 19–20). Finally, the surviving label λ is enheaped (line 21).

As before, the actual simplest near-fastest route can be retrieved with standard backtracking; details are omitted.

Theorem 5. *The SNF-A* algorithm correctly finds a simplest near-fastest route from n_s to n_t.*

Analysis. AllSimplestFastest and AllFastestSimplest require $O(\delta^2|E| + \delta|V|\log|V|)$ amortized time and $O(\delta|V|)$ space. In the worst case, the algorithm may examine all $k = \binom{|E|}{(1+\epsilon)L(\rho^{SF})/\Delta d}$ possible routes from n_s to n_t having at most $(1+\epsilon)L(\rho^{SF})/\Delta d$ edges. Each node of the road network may be assigned up to k labels, one per possible route. The number of enheap and deheap operations equals the number of labels $k|V|$. Furthermore, the number of update operations is equal to k^2 per edge, for a total of $k^2|E|$. Assuming a Fibonacci heap, the time complexity of the traversal is $O(k^2|E| + k|V|\log|V|)$ amortized. Moreover, since the heap may contain an entry for each label, the space complexity is $O(k|V|)$. Overall, the time complexity of SNF-A* is $O(\delta^2|E| + \delta|V|\log|V| + k^2|E| + k|V|\log|V|)$ amortized, while its space complexity is $O(\delta|V| + k|V|)$.

Discussion. Similarly to the case of SNF-DFS, the invocation of the AllSimplestFastest and AllFastestSimplest procedures is not necessary for SNF-A*.

5. EXPERIMENTAL EVALUATION

This section, presents an experiment evaluation of our methodology for Problems 1–4. Section 5.1 details the setup of our analysis. Section 5.2 qualitatively compares the proposed methods, and Section 5.3 studies the scalability.

5.1 Setup

Our experimental analysis involves both real and synthetic road networks. We use the real road networks of the following cities taken from OpenStreetMap: Oldeburg (OLB), Berlin (BER), Vienna (VIE) and Athens (ATH), containing 1,672 roads and 2,383 intersections, 15,246 roads and 25,321 intersections, 20,224 roads and 27,563 intersections, and 76,896 roads and 108,156 intersections, respectively. The weighted average degree of an intersection in these road networks is 2.09, 2.15, 2.17 and 2.19, respectively.

To study the scalability of our methodology we also generated synthetic road networks by populating the OLB road network. The idea is the following. In an attempt to capture the structure of a real network, a synthetic road network is defined as a set of neighborhoods connected to each other through a backbone road network. OLB is used to capture the internal road network of a neighborhood. Figure 2 pictures the 24 intersections used to enter/exit the internal road network of a neighborhood from/to the backbone.

Finally, to construct a backbone network we consider two different topologies. The grid-based topology of degree τ is constructed by 2τ roads, τ^2 intersections, and defines $(\tau-1)^2$

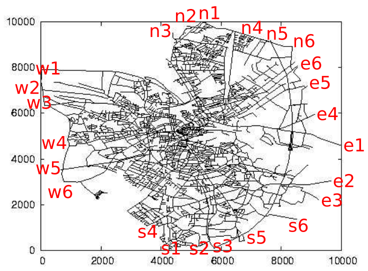
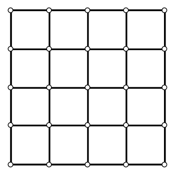


Figure 2: The OLB road network and its 24 entrances/exits.


(a)

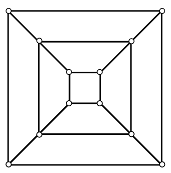

(b)

Figure 3: Examples of backbone networks: (a) grid-based with 10 roads, (b) ring-based with 16 roads.

neighborhoods. On the other hand, a ring-based topology of degree τ is constructed by $4(\tau+1)$ roads, 4τ intersections, and defines $4(\tau-1)+1$ neighborhoods. Figure 3(a) and (b) show an example of a grid-based and a ring-based backbone road network of degrees 5, and 3, respectively. The grid-based backbone consists of 10 roads connected through 25 intersections and defines 16 neighborhoods, while the ring-based backbone consists of 16 roads connected through 12 intersections and defines 9 neighborhoods.

To assess the performance of the routing methods, we measure their average response time and the average number of routes examined over $1,000$ queries. Finally, in case of the simplest near-fastest and the fastest near-simplest route problems, we test the methods varying ϵ inside $\{0.01, 0.05, 0.1, 0.2, 0.3\}$.

5.2 Comparison of Routing Methods

The first set of experiments involves the OLB, BER, VIE, and ATH real road networks with the purpose of identifying the best method for each of the problems at hand.

Table 2 demonstrates the results for the fastest simplest and the simplest fastest route problems. We first observe that FS outperforms BSL by several orders of magnitude. In fact, we managed to execute BSL only on the smallest road network (OLB) due to its extremely high response time. This is expected as BSL needs to enumerate an enormous number of routes to identify the final answer. On the other hand, we observe that FS, SF identify the corresponding routes in less than half a second for all real networks.

Finally, we investigate which is the best method for the fastest near-simplest and the simplest near-fastest route problems. Note that for the purpose of this experiment we include two additional methods termed FNS-A*-WB and SNF-A*-WB. These algorithms follow the same principle as FNS-A* and SNF-A* respectively, without however invoking the AllFastestSimplest and AllSimplestFastest procedures (equivalently they assume $sfL[n] = sfC[n] = fsL[n] = fsC[n] = 0$ for any node n). In addition, note that because of their high response time, we were able to execute FNS-DFS and SNF-DFS only on the smallest road network, OLB. Figure 4 clearly shows that FNS-A* and SNF-A* are the dominant methods for the problems at hand. In fact with the exception of the smallest road network, OLB, they outperform their competitors by at least one order of magnitude. The superiority of FNS-A* (SNF-A*) over FNS-A*-WB (SNF-A*-WB) supports our decision to invoke the AllFastestSimplest and AllSimplestFastest procedures before the actual search takes place.

We also observe that as ϵ increases, the response time of the methods that solve simplest near-fastest route problem decreases. Specifically, the response time of SNF-A*-WB always decreases while the time of SNF-A* first increases and after $\epsilon = 0.1$ or $\epsilon = 0.2$ it drops. Note that this trend is also followed by the average number of routes examined by the methods. The reason for is that the larger ϵ is, the more routes have acceptable length and thus need to be examined. At the same time, however, it is more likely to early identify a candidate answer, which can enhance the pruning mechanism and thus accelerate the query evaluation.

5.3 Scalability Tests

In the last set of experiments we study the scalability of the best methods identified in the previous section, i.e., FS, SF, FNS-A* and SNF-A*. For this purpose, we generate synthetic road networks varying the degree of the topology τ, and thus, the size of the road network. Particularly, for a grid-based backbone network τ takes values inside $\{2, 3, 4, 5\}$, while for a ring-based backbone inside $\{1, 2, 3, 4\}$. Figure 5 reports on the scalability tests. As expected, the response time of all methods increases when the degree of the topology increases. Even for the expensive fastest near-simplest and the simplest near-fastest route problems, our methods always identify the answer in less than half a second for $\epsilon = 0.1$. Although we do not include figures for other values of ϵ, our experiments show that this holds for every other combination of τ and ϵ.

6. CONCLUSION

This paper dealt with finding routes that are as simple and as fast as possible. In particular, it studied the fastest simplest, simplest fastest, fastest near-simplest, and simplest near-fastest problems, and introduced solutions to thems. The proposed algorithms are shown to be efficient and practical in both real and synthetic datasets.

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7. REFERENCES

[1] I. Abraham, D. Delling, A. V. Goldberg, and R. F. Werneck. A hub-based labeling algorithm for shortest paths in road networks. In *Experimental Algorithms*, pages 230–241. Springer, 2011.

[2] R. Bauer and D. Delling. Sharc: Fast and robust unidirectional routing. *Journal of Experimental Algorithmics (JEA)*, 14:4, 2009.

[3] R. Bauer, D. Delling, P. Sanders, D. Schieferdecker, D. Schultes, and D. Wagner. Combining hierarchical and goal-directed speed-up techniques for dijkstra's algorithm. *Journal of Experimental Algorithmics (JEA)*, 15:2–3, 2010.

[4] T. H. Byers and M. S. Waterman. Determining all optimal and near-optimal solutions when solving shortest path problems by dynamic programming. *Operations Research*, 32(6):1381–1384, 1984.

[5] T. Caldwell. On finding minimum routes in a network with turn penalties. *Communications of the ACM*, 4(2):107–108, 1961.

[6] D. Delling, A. V. Goldberg, A. Nowatzyk, and R. F. Werneck. Phast: Hardware-accelerated shortest path trees. *Journal of Parallel and Distributed Computing*, 2012.

[7] E. W. Dijkstra. A note on two problems in connexion with graphs. *Numerische mathematik*, 1(1):269–271,

Table 2: Real road networks: performance analysis for solving Problems 1 and 2.

	BSL		FS		SF	
road network	**Response time (sec)**	**Routes examined**	**Response time (sec)**	**Routes examined**	**Response time (sec)**	**Routes examined**
OLB	68.7	121, 236, 000	0.003	2286.82	0.003	2418.35
BER	–	–	0.055	27226.3	0.040	27611.7
VIE	–	–	0.057	29301.8	0.042	29250.8
ATH	–	–	0.346	117, 973	0.207	120, 329

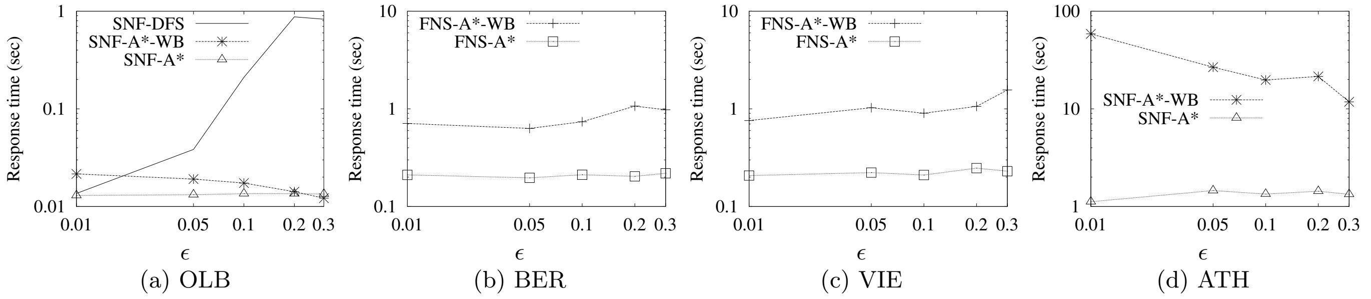

Figure 4: Real road networks: performance analysis for solving Problems 3 and 4.

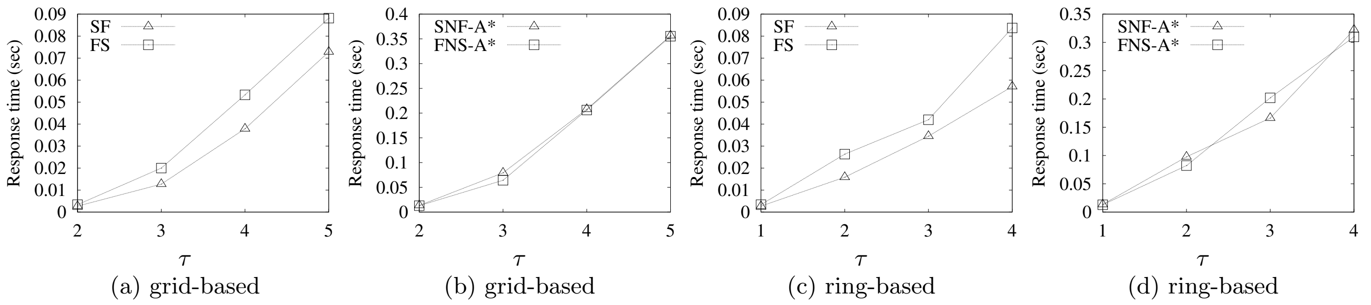

Figure 5: Synthetic road networks: scalability tests for $\epsilon = 0.1$.

1959.

[8] M. Duckham and L. Kulik. "simplest" paths: Automated route selection for navigation. In *Conference On Spatial Information Theory (COSIT)*, pages 169–185, 2003.

[9] R. Geisberger, P. Sanders, D. Schultes, and D. Delling. Contraction hierarchies: Faster and simpler hierarchical routing in road networks. In *Experimental Algorithms*, pages 319–333. Springer, 2008.

[10] R. Geisberger and C. Vetter. Efficient routing in road networks with turn costs. In *Experimental Algorithms*, pages 100–111. Springer, 2011.

[11] A. V. Goldberg and C. Harrelson. Computing the shortest path: A search meets graph theory. In *Proceedings of the sixteenth annual ACM-SIAM symposium on Discrete algorithms*, pages 156–165. Society for Industrial and Applied Mathematics, 2005.

[12] R. J. Gutman. Reach-based routing: A new approach to shortest path algorithms optimized for road networks. In *ALENEX/ANALC*, pages 100–111, 2004.

[13] B. Jiang and X. Liu. Computing the fewest-turn map directions based on the connectivity of natural roads. *International Journal of Geographical Information Science*, 25(7):1069–1082, 2011.

[14] H.-P. Kriegel, M. Renz, and M. Schubert. Route skyline queries: A multi-preference path planning approach. In *ICDE*, pages 261–272, 2010.

[15] W. Matthew Carlyle and R. Kevin Wood. Near-shortest and k-shortest simple paths. *Networks*, 46(2):98–109, 2005.

[16] R. H. Möhring, H. Schilling, B. Schütz, D. Wagner, and T. Willhalm. Partitioning graphs to speedup dijkstra's algorithm. *Journal of Experimental Algorithmics (JEA)*, 11:2–8, 2007.

[17] T. A. J. Nicholson. Finding the shortest route between two points in a network. *The Computer Journal*, 9(3):275–280, 1966.

[18] A. Raith and M. Ehrgott. A comparison of solution strategies for biobjective shortest path problems. *Computers & Operations Research*, 36(4):1299–1331, 2009.

[19] D. Sacharidis and P. Bouros. Routing directions: Keeping it fast and simple. *CoRR*, abs/1309.4396, 2013.

[20] P. Sanders and D. Schultes. Highway hierarchies hasten exact shortest path queries. In *Algorithms–Esa 2005*, pages 568–579. Springer, 2005.

[21] F. Schulz, D. Wagner, and C. Zaroliagis. Using multi-level graphs for timetable information in railway systems. In *ALENEX*, pages 43–59. Springer, 2002.

[22] S. Winter. Modeling costs of turns in route planning. *GeoInformatica*, 6(4):345–361, 2002.